

The Value of Fairness: Trade-offs in Repeated Dynamic Resource Allocation

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Abstract—Resource allocation problems are an important part of many distributed autonomous systems. In sensor networks, they determine which nodes get to use the communication links, in SmartGrid applications they decree which electric vehicle batteries are loaded, and in autonomous power management they select which generators produce the power required to satisfy the overall load. These cases have been considered in the literature before under the aspect of demand satisfaction: how well can distributed algorithms with local knowledge approximate the best allocation. A factor that has been ignored, however, is fairness: how fair is the resource allocation and—in extension—the distribution of revenue, wear, or recovery time.

In this paper, we bring together previously disjoint approaches on dynamic distributed resource allocation and on fairness in electronic institutions. We show that fair allocations based on Ostrom’s principles and on Rescher’s canons of distributive justice create value in repeated resource allocations. We apply the scheme to solve the multi-objective problem of distributing load to generators fairly based on demands made by the individual generators. Our evaluation shows that a fair distribution increases satisfaction of the individual agents while reducing the hazard of optimising the problem in the short-term at the cost of long-term robustness and stability.

I. INTRODUCTION

Resource allocation problems usually use objective functions that optimise resource usage, utility or overall cost. Such an approach is acceptable as long as the participants in the allocation depend on the resources and have no alternative but to be part of the scheme. In contrast, in systems that depend on the voluntary participation of autonomous entities and their provision of resources, the situation to be addressed is different: we are now dealing with multi-criteria optimisation, where additional considerations include the retention of as many participants as possible, and ensuring the long-term endurance of the system itself.

The distributed resource allocation scheme TruCAOS [1] is an exemplar of such a system. It was developed to create schedules in autonomous power management systems. In a market-based scheme, load created by power consumers is assigned to controllable prosumers. The term “prosumer” stands for producers as well as consumers and includes generators based on sources such as coal, biofuel, gas, as well as electric vehicles and other storages. Each prosumer can make bids on a share of the overall load. The best bids are selected by the share of the load they cover and the price they incur. While this approach provides excellent results with respect to the distribution of the load and the overall cost, situations can

occur in which specific prosumers always win the bidding process. Other prosumers participating in the scheme thus are not able to achieve any monetary rewards for their participation, effectively driving them out of the system.

This can be highly problematic. Power plant scheduling is a repeated process that occurs at certain instances to schedule energy production for specific intervals. The schedules are highly dependent on the production of stochastic sources such as solar or wind power plants. If the input of these sources decreases unexpectedly while the demand surges, situations can occur in which a cohort of ‘cheaper’ power plants can no longer satisfy the demand. In such cases, additional degrees of freedom are required to resolve the ‘extreme’ condition, in practice requiring that a larger number of producers participate in the scheme than is strictly necessary for ‘normal’ conditions. If this ‘slack’ in the system has been driven out because of price pressure, no capacity to compensate for the sudden surge is available.

In this case, the achievement of maximum utility has come at the cost of increased exposure to risk, a lack of robustness and possibly complete systemic failure. We would contend that the exclusive emphasis on utility has driven out considerations of *fairness*. It has been shown that fairness does indeed come at a *price* [2], as a reduction of overall utility; however, in this paper, we are concerned with the *value* of fairness, i.e. that some ‘inefficiency’ in the system is tolerable in return for increased satisfaction. This in turn provides the basis for less-easily quantifiable properties (or non-functional requirements) such as stability, robustness and endurance.

In the ongoing transition to open, self-organising distributed computing systems and networks, we believe that the resource allocation problem found in TruCAOS will increasingly occur. Examples include sensor networks that determine which nodes get to use the communication links, smart grid applications that decide which electric vehicle batteries are charged and discharged, cloud computing solutions that need to balance total cost of ownership versus quality-of-service constraints, grid computing, and so on. All of these systems exhibit features of voluntary, opportunistic and dynamic assembly and disassembly, shared use of endogenous resources, repeated (rather than one-off) scheduling of resource provision, an economy of scarcity rather than excess, and the possibility of non-compliant behaviour.

In this paper, we develop an algorithmic framework based on an innovative synthesis of [3], which proposed a self-

organising resource allocation scheme based on Rescher's theory of distributive justice, and issues of cooperation and trust in resource allocation in autonomous power management systems [1]. This provides the basis for the implementation of an experimental testbed which is used to explore the trade-off between optimising utility and satisfaction with respect to self-organised repeated resource allocation. We discuss how fair solutions (using a specific fairness metric) give agents an incentive to stay in the system, thus providing redundancy that increases resilience and the ability to deal with critical situations. As a result, short-term losses in optimality with respect to efficiency or economic considerations can lead to long-term benefits with respect to stability and robustness.

The paper is therefore structured as follows: Section II introduces the case study on autonomous power management outlined above in more detail and provides background on fairness in electronic institutions. The concepts from electronic institutions are then mapped to the case study in Section III, leading to the fair resource allocation scheme described in Section IV. We illustrate the results of the evaluations in Section V, along with a detailed discussion of the trade-offs of fairness and costs that we observed. While the experimental results do not put exact prices on these long-term benefits, we show that such trade-offs exist. We should therefore be aware of them, or better still, use them to our advantage.

II. AUTONOMOUS POWER MANAGEMENT SYSTEM, ELECTRONIC INSTITUTIONS AND THE LPG'

This section will introduce the case study used in the following, the theoretical background on electronic institutions based on Ostrom's principles and Rescher's cannons, and the variant of the Linear Public Goods game used in the analysis of the resource allocation problems we are concerned with.

A. Autonomous Power Management (APM)

One of the main tasks of a power management systems is scheduling, i.e., the determination of the output levels of dispatchable prosumers¹ for future time steps considering physical as well as economical constraints. Automating this process promises benefits in adaptivity, costs, and especially in the ability to include a large number of distributed prosumers that have so far been largely ignored. This is especially desirable as the need for complex distribution networks is decreased and complications such as voltage band constraints [4] can be alleviated. Such a scheme, however, requires prosumers to act as agents that proactively participate in the creation of schedules and in maintaining grid stability (see, e.g., [5], [6]).

The major disadvantage of such an automation is the enormous computational complexity incurred by scheduling thousands of dispatchable prosumers based on the status of the electric grid. Scheduling takes place in fifteen minute intervals, limiting the time each decision can take. Since the problem is NP-hard, scalability becomes a major concern for systems of realistic size. Even municipal utilities have to deal with thousands of potentially dispatchable prosumers in their catchment area. One proposed solution to this problem is the

¹Later on in the paper, we will use the term "producer". Please note that many producers can also create a load (e.g., storages). In this section, we stick to "prosumer" to include dispatchable loads such as refrigerating warehouses.

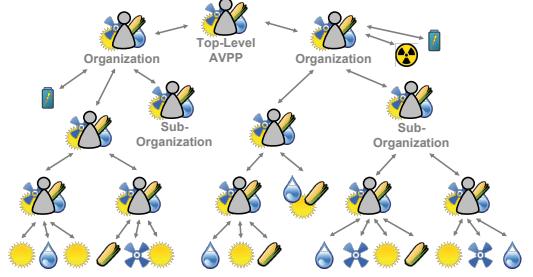


Figure 1. Hierarchical system structure of a future autonomous power management system: prosumers are structured into systems of systems represented by AVPPs, thereby decreasing the complexity of control and scheduling. AVPPs can be part of other AVPPs and can represent organisations.

introduction of intermediaries that form a hierarchy [7] of so-called *Autonomous Virtual Power Plants* (AVPP) [8] as shown in Figure 1. Each AVPP deals with a smaller sub-problem, thus regionalising scheduling and making it possible to scale the system to the number of agents required. The structure of an AVPP is dynamic and agents can enter and leave the AVPP at their own discretion.

Within an AVPP, schedules can be created in a centralised or in a distributed fashion. For centralised approaches (see, e.g., [9], [10]), the intermediary must know the physical and economic limitations of the prosumers and combine them in a model [11]. In distributed schemes, prosumers can keep their control models private and do not have to disclose potentially classified information. Due to privacy concerns, cooperation-based approaches, such as TruCAOS mentioned in the introduction or the scheme put forward in this paper (cf. Section IV), allow prosumers to formulate valid schedules without disclosing their internal models.

An important aspect that has to be considered in all such scenarios is the inherent uncertainty of electricity consumption and production. The main factor in the creation of schedules is the *residual load* that combines forecasts of the production of intermittent power sources (mainly solar and wind, but also domestic heat and power) with forecasts of electricity consumption. Basically, only the consumption that will not be covered by intermittent power sources has to be distributed to dispatchable prosumers. Since both consumption and intermittent production are influenced by external factors, the forecasts are inaccurate. When creating schedules, this inaccuracy has to be taken into account. In addition, a prosumer that promises to provide a certain power and fails to deliver might destabilise the system. If such behaviour is observed repeatedly, it should be taken into consideration in the scheduling.

This can be achieved by using the notion of trust, i.e., a measure of the expectation that an agent sticks to its promises. A promise in this context is a forecast of a certain production or consumption. The trust value—in this case a credibility value—then gives an estimate of the long-term deviation from these forecasts that can be used to calculate the *expected residual load* and the expected production respectively. Another form of trust—reliability—is calculated by measuring the time a power plant is connected to the power grid, a value that can be influenced by unexpected downtimes or failures. These measures can be used in electronic institutions as the basis for decentralised control decisions as outlined in the next section.

B. Enduring Electronic Institutions

An AVPP in the form discussed above constitutes an electronic institution that embodies an open, embedded, resource-constraint system with decentralised control over resources and the expectation of both intentional and unintentional errors. Pitt et al. [12] state that in such systems the ‘optimal’ distribution of resources (in an utilitarian sense) is less important than the ‘robustness’ or ‘survivability’, hence the endurance, of the distribution mechanism. In case of the AVPP, this endurance depends on the availability of prosumers providing dispatchable power to deal with unexpected surges.

Electronic institutions based on the principles for enduring self-governing institutions, as introduced by Elinor Ostrom, and on design principles for allocation systems incorporating distributive justice, based on the work of Nicolas Rescher, provide the means to ensure this endurance. When using an appropriate scheme adhering to these principles, the system becomes indeed survivable and robust based on the continued willingness of all participants to contribute to the system [3].

In a Common-pool Resource (CPR) allocation system, the divisible pooled resource is public but restricted in quantity, while the system components ‘are required to share and appropriate resources in order to satisfy individual goals’. The resources may either be exclusively exogenous (provided by an external source) or endogenous (provided by the system itself) or a mixture of both. Game theory predicts the *Tragedy of the Commons* [13] which describes the dilemma of rational agents that are dependent on a limited CPR, acting independently and rationally according to each one’s self-interest in allocating such an amount of resources that the common pool is depleted in the short-term, even though such a behaviour contradicts both the group’s and the individual agents’ long-term interest. The depletion of the resource renders depending individuals or organisations unable to operate further. In economics this phenomena can be found in a large number of systems, tightly connected to the concept of endurance and sustainable development, like fishery, water irrigation, farming, with respective *commons* fish, water and soil or meadows [14].

Ostrom observed, however, that delegating the government of commons to *institutions* avoids the tragedy of the commons. Her notion of institution describes a set of working rules, regulating and constraining provision to and appropriation from the resource. These rules are used ‘to determine who is eligible to make decisions in some *arena*, what actions are allowed or constrained, what aggregation rules will be used, what procedures must be followed, what information must or must not be provided, and what payoffs will be assigned to individuals dependent on their actions’ [14, p. 51].

According to Ostrom [14, p. 90], eight design principles ensure the endurance of self-governing institutions. The most important ones are: P1) clearly defined boundaries of who has rights to appropriate resources; P2) congruence between appropriation and provision rules and local conditions; P3) those affected by the operational rules participate in the selection and modification of those rules (collective-choice arrangements); P5) graduated sanctions for violating rules and P8) layered or encapsulated CPR, with local CPR at the base level.

These design principles do not, however, give details on the allocation mechanisms that should be used, especially

concerning the fairness of the outcome for the participants. Such details can be found in the field of *distributive justice*, especially in the work of Nicholas Rescher. He proposes to assess fairness based on the *seven canons of distributed justice* [15]. These can be used as ultimate determinant of individual claims, e.g., to resolve if participants are treated as equals or according to their needs. Rescher also concludes that justice is not ensured by valuing the individual claims on basis of only one canon, as they are monoistic, each recognising but one solitary mode of claim production. Instead, he states that distributive justice consists of the pluralistic *Canon of Claims* which treats people according to all their legitimate claims, positive and negative, i.e., according to a valuation of all canons [15, p. 81f]. He also concludes a need for individualised canon selection within the *Canon of Claims* based on the context of the allocation system.

C. Fair Distributed Resource Allocation: The LPG’

The characteristics of open systems, such as autonomous power management, the principles put forward by Ostrom, and the canons proposed by Rescher are combined in the LPG’, a variant of the Linear Public Goods Game (LPG) [3]. The LPG is a classical experiment in game theory, demonstrating the prominent contradiction of following the dominant individual strategy of withholding one’s own resources to the disadvantage of all players, while the maximum utility could be achieved when fully provisioning the resources of all players. Each player has resources that it can contribute to the common pool and that it requires from the pool. The LPG’ assumes an economy of scarcity, meaning that the total amount of required resources exceeds the amount available resources. The game is used to establish a resource allocation system for open systems ensuring fairness and endurance, gaining a better balance of utility and fairness, for compliant agents, and hence improved stability.

The LPG’ is repeatedly played in clusters of agents, where a game step consists of the following sequence of actions:

- 1) players determine the resources available for contribution to the public pool
- 2) players determine their actual need for resources
- 3) players make a public demand for resources
- 4) players announce how many resources they are willing to provide to the public pool
- 5) players are informed of the allocation of resources, i.e., how many resources they can appropriate
- 6) players appropriate resources from the public pool (not necessarily identical to appropriation)

The assumed resource scarcity raises incentives not to contribute at all and to violate rules in order to increase the amount of appropriated resources. Cheating is possible in two places in the system: 1) publishing a higher demand than internally determined, and 2) appropriate more than got allocated. Monitored non-compliant agent behaviour is fed back to the allocation scheme, which considers and sanctions the misbehaviour in the next rounds by reducing the resource allocation. Thus, compliance to rules is rewarded while cheating is penalised by reducing the rank of the perpetrator’s claims in the next round.

A major extension to the LPG is the introduction of satisfaction of the individual agent. It is measured by assessing the deviation of published demand and actual allocation. (Dis-) Satisfaction accumulates through reinforcement over the game rounds and causes the agent to abandon the cluster when the satisfaction falls below a certain threshold. The **allocation scheme** thus plays a crucial role for an agent's satisfaction. The most trivial, though possibly unfair, scheme is a uniform allocation of the public pool resources to the agents. Pitt et. al [3] showed that a fair allocation, implemented by Rescher's canons of distributive justice increases utility and satisfaction in the long-term. For this purpose, the canons are represented by an agent's *legitimate claims*. These claims are assessed by functions that are used to calculate ranking lists based on the agent's merits. The set of functions (F) in the LPG' is defined in [3, p. 3f], considering, *inter alia*, an agent's demand, appropriation, compliance and satisfaction.

To implement the canon of claims, i.e., to combine the individual claims, the *Borda count voting protocol* is used. Each canon represents a preferential voter $f_* \in F$ [3], with the agents being the candidates to vote for. Each voter creates a rank list, by computing the rank of each agent. Every rank list assigns Borda points to an agent proportional to its rank. The Borda points of an agent from each ranking list are aggregated to a resulting total Borda score. In order to reconcile conflicts between the claims, each of them is assigned a weight representing its importance. The aggregation of Borda points is then computed as a weighted sum. Then, a queue containing the agents in decreasing order of the total Borda score is created and resources are allocated to the agents in decreasing order of the queue. Each agent receives the full demanded quantity until the resource pool is depleted. The canon weights are determined by the agents themselves based on collective-choice-rules of the self-organising electronic institution that plays the LPG'. They are updated at the end of a round.

III. MAPPING: ELECTRONIC INSTITUTION & APM

In this section we present the mapping of concepts from electronic institutions to the case study of the APM, where the allocation in the scheduling process is delegated to an enduring electronic institution, incorporating fairness and hence promoting stability and endurance.

Differences in the conceptual notions and design principles of the LPG' and the APM are presented in Table I and described subsequently. The LPG' is designed as a general approach, without immediate real-world application intention. Thus, its system nature is abstract, as are its players. There is neither individualisation nor differentiation in the player model

Table I. CONCEPT MAPPING: ELECTRONIC INSTITUTION AND APM

Concept	Elec. Inst. (LPG')	APM
System nature	abstract	cyber-physical (real-world), mission-critical
Player	homogeneous abstract player	heterogeneous power plant
Institution	abstract 'cluster'	macro level: APM; meso level: AVPP
Resource type	abstract resource	power demand (load)
Resource origin	endogenous	exogenous
Res. economy	scarcity	scarcity/surplus (each infeasible or feasible)
Player demands	arbitrarily, time-independent	static, time-dependent

— they are homogeneous. The institutional boundaries are defined through memberships to abstract clusters. In contrast, the APM can be classified as a Cyber-Physical System (CPS), as its computational elements control physical entities [16]. The APM defines an institution itself on the macro level, whereas an AVPP constitutes an institution on the meso level. Its actors, i.e., the players, are power plants, which are classified by their energy source where each class exhibits distinct characteristics. Furthermore, a power plant has individual properties, such as the maximum power output. Players in the APM are thus highly heterogeneous. We consider both the technical and economical context of power management systems, which impose strict requirements and exhibit distinct properties the allocation system has to satisfy:

- 1) Since power plants are physical components, physical restrictions such as maximum power production and change rate (slope) are to be treated as hard constraints and must not be violated by the scheduling.
- 2) Power plants are to be considered as market participants, which include economical factors like optimal power production in the assessment of their perceived fairness.

Employing agent autonomy in a mission-critical context constitutes an inherent conflict. While the focus in a CPS lays on control and optimization, satisfying hard constraints, unrestricted autonomy of agents may jeopardise the system goals, e.g., when players arbitrarily leave the system or behave maliciously. On the one hand we thus argue to limit agent autonomy by enforcing compliance, i.e., forbid cheating, and using hard constraints. On the other hand, we sacrifice the optimality of the schedules w.r.t. load distribution to both promoted agent-autonomy and degree of self-organisation as well as agent satisfaction and fairness. We discuss the implications of this trade-off in Section VII. The systems' substantial difference in the level of abstraction is also reflected in the allocation system with respect to the type of resource. In case of the LPG' the resource is only regarded as an abstract concept, whereas in the APM it is defined as the load, i.e., the power demand, the power plants are in competition to fulfil. In contrast to the LPG' the resource is not provided by the agents themselves in the APM but is assigned from an instance outside of the institution and is hence of exogenous nature. The assumption of an economy of scarcity regarding the resources in the LPG' does not generally hold in the APM, where an economy of scarcity would mean that an AVPP's total optimal power production would exceed its assigned load, such that there would be a competition between the producers to satisfy this load. The demand of a controllable power plant (CPP) will converge to its static optimal production. Based on the CPP's current production, its slope may bound the demand which is thus time-dependent. In the LPG', players are assigned resources randomly at the beginning of each round. They determine their need based on this assignment.

Figure 2 gives a high-level overview of the target system. An AVPP gets load assigned from its superordinate AVPP and distributes this load to its constituents within the scheduling process. The scheduling utilises an electronic institution to perform the resource allocation. Due to the inherent coupling of an AVPP and its constituents, the borders of the organisation they form and the role of an appropriator are clearly defined, as demanded in Ostrom Principle P1.

Within the electronic institution, resource distribution is performed, based on a modified LPG' with APM-specific canons which, along with the constituent's demand, determine the allocations for the constituents. The delegation of scheduling to encapsulated AVPPs satisfies P8. A CPP receives a *load allocation*, i.e., a resource allocation, as the outcome of the scheduling of its AVPP, appropriates load and produces power, matching the appropriated load which it feeds into the power grid. Deviations from the allocated and appropriated load and from the appropriated and produced power are subsumed as *operational deviation*. Operational deviations are measured by a suitable trust metric and are fed back to the allocation system in form of a canon, sanctioning misbehaviour or unreliability.

Based on its power plant model, a CPP determines and publishes a demand for resources to the allocation system. However, depending on the resource situation (*resource case*), i.e., the total quantity of available compared to demanded resources, it is likely that a deviation between the quantity of demanded and allocated resources, expressed as *distributional deviation*, exists. This deviation is quantified in different ways, since it is the basis for the CPP satisfaction metric.

The weight determination in the electronic institution of the LPG' stipulates a self-organisation process incorporating voting and thus following Ostrom principle P3 (collective choice-agreements).

As denoted by *feedback* in Figure 2, a CPP's demand, distributional deviation, satisfaction and trust are published to the allocation system to be evaluated as claims. Since the distributional deviation and thus also the satisfaction, is an indirect result of a load allocation, i.e. a scheduling outcome, and is used in claims to determine the allocation, i.e. the schedule, this represents a feedback loop.

IV. FAIR DYNAMIC ALLOCATION IN THE APM

In this section we present details of the implementation of fair dynamic scheduling. Figure 3 gives an overview of the APM allocation system, whose action sequence is outlined as follows:

- 1) At the beginning of each round each CPP i within an AVPP's constituting set of power plants CPP determines its demand d_i (see Section IV-A) and publishes it.
- 2) The AVPP's allocation scheme in the scheduling process determines the prevailing resource case based on its as-

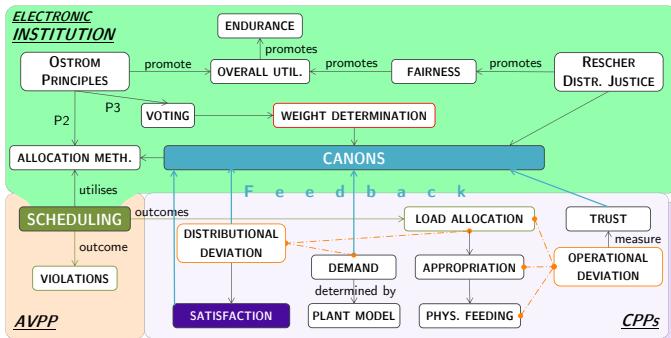


Figure 2. Semantic map of the final system: concepts, involved organisations, their relations and interactions.

signed load ($load_{res}$) and published demands of its CPPs (see Section IV-B), followed by computing an allocation a_i for each $i \in CPP$ (see Section IV-C).

- 3) Each CPP i receives an allocation a_i and appropriates $a'_i = a_i$, as cheating is not allowed.
- 4) In the last step each CPP assesses its satisfaction σ_i^C with its allocation (see Section IV-D).

A. Demand Determination, Deviation and Satisfaction Metrics

For the determination of the demand $d_i(t)$ for a CPP i at time step t , the hard constraints for minimum production, maximum production, and slope of the CPP must not be violated. Let p_i^{min} denote the absolute minimum production of i and p_i^{max} its absolute maximum production. The slope s_i bounds the output of i , based on its current output $p_i^{prod}(t)$, such that there exist time-dependent minimum and maximum production bounds, denoted as $p_i^{min}(t)$ and $p_i^{max}(t)$ respectively.

The static optimal production of i is given by i 's model and shall be denoted by p_i^* . Since every CPP strives to reach its optimum, $d_i(t)$ converges to p_i^* , but is bounded by $p_i^{min}(t)$ or $p_i^{max}(t)$ and therefore depends on the current output. After all CPPs published their demands, they get an allocation from the APM allocation scheme and assess their satisfaction. Starting from the trivial demand-allocation deviation $\delta_i(t) = a_i(t) - d_i(t)$, $\delta_i(t)$ is normalised with respect to i 's time-dependent minimum and maximum production bounds for comparison with other CPPs:

$$\delta_i^n(t) = \frac{a_i(t) - d_i(t)}{p_i^{max}(t) - p_i^{min}(t)} \quad \text{and} \quad \delta_i^n(t) \in [-1; 1] \quad (1)$$

The economic context of the plants raises the need for a sophisticated deviation metric, where, e.g., positive and negative deviations from the optimum shall have distinctive implications, i.e., power plant operators prefer over-production to under-production of the same quantity. We thus model the deviation metric asymmetrically, weighting positive and negative deviations differently by introducing a factor $\omega^+ \in [0; 1]$ which possibly attenuates positive $\delta_i^n(t)$ in the resulting weighted deviation $\delta_i^w(t)$ as follows:

$$\delta_i^w(t) = \begin{cases} \delta_i^n(t) \cdot \omega^+ & \text{if } \delta_i^n(t) \geq 0, \\ \delta_i^n(t) & \text{otherwise} \end{cases} \quad (2)$$

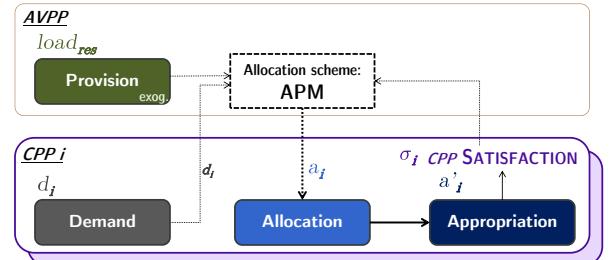


Figure 3. The APM allocation system: within the scheduling of an AVPP, its CPPs publish a demand, get an allocation from the allocation scheme, appropriate their allocation and assess their satisfaction with the outcome.

B. Resource Case and Determination of Allocator Sequence

In step 2, the given residual load is fully allocated to the CPPs. For this purpose, the APM allocation scheme determines the prevailing resource case by comparing the exogenous resource provision, i.e., $load_{res}(t)$, with the published demands and both maximum and minimum production capacities. It then determines the individual allocations according to this resource case, thus implementing Ostrom's principle P2.

Infeasible resource cases cause a violation $v(t) = \sum p_i^{prod}(t) - load_{res}(t)$, as either the complete residual load can not be fulfilled ($v(t) < 0$: under-production) or there is an over-production ($v(t) > 0$). Feasible (exact) cases have $v(t) = 0$. We informally enumerate the resource cases as follows:

- 1) **Upper bound** All power plants are allocated their maximum output (feasible or infeasible)
- 2) **Lower bound** All power plants are allocated their minimum output (feasible or infeasible)
- 3) **Variable** There is a variability in the allocation: CPPs get individual allocations within their feasible production range (feasible; scarcity: $load_{res}(t) > \sum d_i(t)$; surplus: $load_{res}(t) < \sum d_i(t)$, exact: $load_{res}(t) = \sum d_i(t)$)

The APM allocation scheme implements a set of particular mechanisms for resource allocation (*allocators*), to cover specific resource case aspects. Depending on the resource case, a single allocator or an adaptive sequence of allocators is used to define the ultimate allocations. Thus we refine $a_i^*(t)$ to denote a single allocation for CPP i from allocator $*$ at time t and we let $A_i(t) = \sum_* a_i^*(t)$ denote the sum over all allocations of i . The mapping of allocators for the resource case and the determination of the allocator sequence—both part of the APM allocation algorithm—are shown in Figure 4. In case of the *upper bound* resource cases, the **MAXall** allocator allocates resources equal to the CPPs' time-dependent maximum production quantity ($A_i(t) = a_i^{MAXall}(t) = p_i^{max}(t)$, $\forall i \in CPP$). Analogously, the **MINall** allocator allocates time-dependent minima ($p_i^{min}(t)$) for the lower bound resource cases. The algorithm in both cases terminates after that single allocation.

For the *variable* resource cases, a sequence of allocators is executed. The allocation quantity of each allocator is bounded by the maximum production constraint. After each allocation, the residual load as well as minimum and maximum production constraints are updated, considering the quantity of allocated resources. In all *variable* subcases **MINall** is allocated first, as a base allocation to satisfy the plants' minimum production constraint. In the *variable scarcity* case, where the CPPs are in competition over resources, i.e., there is an economy of scarcity, the **APM LC** allocator, implementing a fair legitimate-claims based allocation (see IV-C), is used. In the *variable exact* resource case, where the residual load equals the sum of demands, the **DEMANDall** allocator allocates resources such that the total allocated resources, including those from **MINall** before, equal the CPPs' demand ($A_i(t) = d_i(t) \forall i \in CPP$). The algorithm then terminates. For the *variable surplus* case, **DEMANDall** is applied. As any further allocation is then detrimental for a CPP because its demand is already fulfilled, the **APM LC** allocator is applied in *inverse mode*, as described in IV-C. It is possible that in the **APM LC**, a CPP's merit based on its claims may be high enough to get an allocation quantity

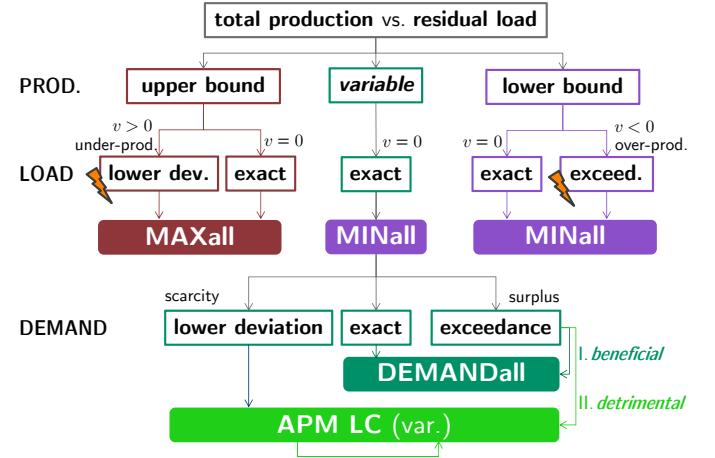


Figure 4. Resource cases and allocator sequence mapping. Unfilled boxes represent resource conditions, filled boxes represent allocators.

which would exceed its maximum production bound. The exceeding resource quantity is then accrued in a resource pool which is (repeatedly) distributed by a new APM LC instance, until this pool is depleted and the algorithm terminates.

C. The APM Legitimate Claims Allocator

The APM LC allocator allocates resources proportional to a CPP's demand and its legitimate claims, valued in form of particular canons of distributive justice for APM resource allocation. As the power plants are highly heterogeneous, we pay special attention to equalise the treatment of CPPs by adequately comparing individual CPPs. For instance, both the demand and maximum production of different CPPs may differ by several orders of magnitude, e.g., when comparing a nuclear power plant to a small community-driven bio-gas power plant. We thus define the relative demand $d_i^r(t) \in [0; 1]$:

$$d_i^r(t) = \frac{d_i(t)}{\sum_{j \in CPP} d_j(t)} \quad (3)$$

We use canons to determine rank lists, reflecting the power plants' relative merits, analogously to the LPG', but based on APM specific canons, as presented in Table II. As the allocations are dependent on the resource case, which differs in the APM for different rounds, the canons operate on data from previous rounds. The computation of the ranking lists and resulting total Borda score $B(i, F)$, analogously to the LPG', is the first step of the APM LC allocator. In *inverse mode*, where allocations are considered detrimental, the ranking lists are reversed, transforming positive to negative claims and vice versa.

The main purpose of the APM LC is to determine allocations proportional to both relative demand d_i^r and total Borda score $B(i, F)$ of CPP i based on the residual load. The relative demand is incorporated in the proportional allocation, as a compensation for the equalisation within the equality canon, considering the CPP heterogeneity. The bounded allocation of APM LC for i is computed as follows (time indices omitted for brevity, ticks depict consideration of allocations from other allocators):

Table II. APM-SPECIFIC CANONS OF DISTRIBUTED JUSTICE

*f*₁ **The canon of equality** equalises heterogeneous CPPs by firstly ranking them in decreasing order of their average absolute weighted deviation $|\delta_i^w|$, secondly ranking in increasing order of their satisfaction σ_i^C , and thirdly in increasing order of the number of rounds in which they received an allocation.

*f*₂ **The canon of needs** ranks the CPPs in decreasing order of their average relative demand d_i^r .

*f*₃ **The canon of productivity** ranks the CPPs in decreasing order of their reliability trust value.

- **The canon of effort** is considered conceptually inapplicable in the APM context and is thus unrepresented.
- **The canon of social utility** ranks the CPPs in decreasing order of their credibility trust value.
- **The canon of supply and demand** is considered conceptually inapplicable in the APM context and is thus unrepresented.
- **The canon of merits and achievements** is considered conceptually inapplicable in the APM context and is thus unrepresented.

$$a_i^{LC} = \text{Min} \left[\underbrace{p_i'^{\max}}_{\text{upper bound}}, \underbrace{\text{load}'_{\text{res}} \cdot (\omega_d \cdot d_i^r + \omega_{lc} \cdot B^n(i, F))}_{\text{proportional allocation}} \right] \quad (4)$$

where ω_d and ω_{lc} are weights $\in [0; 1]$, for the relative demand and the legitimate claims respectively, which allows balancing, and $B^n(i, F)$ is the normalised total Borda score given as:

$$B^n(i, F) = \frac{B(i, F)}{\frac{n(n+1)}{2}}, \text{ where } n = |\text{CPP}| \quad (5)$$

where the divisor states the total sum of Borda points over all voters (canons) and participants (CPPs).

D. Assessment of Satisfaction

Finally, a CPP i assesses a subjective satisfaction $\sigma_i^C(t) \in [0; 1]$ at time t . ϵ^+ and ϵ^- span a skew ϵ -area around the demand. i adapts its satisfaction as follows:

$$\sigma_i^C(t+1) = \begin{cases} \sigma_i^C(t) + \alpha \cdot (1 - \sigma_i^C(t)) & \text{if } \epsilon^- \leq \delta_i^w(t) \leq \epsilon^+ \\ \sigma_i^C(t) - \beta \cdot \sigma_i^C(t) & \text{otherwise} \end{cases} \quad (6)$$

where α and β are coefficients $\in [0; 1]$ which determine the rate of reinforcement of satisfaction and dissatisfaction respectively.

On the meso level, we compute the satisfaction of an AVPP a at t as the average CPP satisfaction over its set of constituting controllable power plants and possibly nested AVPPs CPP_a :

$$\sigma_a^A(t) = \frac{1}{|\text{CPP}_a|} \sum_{i \in \text{CPP}_a} \sigma_i^C(t). \quad (7)$$

V. EMPIRICAL VALIDATION OF APM CPR SCHEDULING

Previously introduced concepts are empirically evaluated in this section by an analysis of scheduler performance with respect to the APM's macro level goals and the expected promotion of both fairness and satisfaction. The results are then compared to scheduling based on constraint satisfaction and optimization (CSOP). We discuss trade-offs of fairness and performance in light of the results obtained from the experiments.

A. Evaluation Methodology & Parameter Configuration

We implemented a testbed, using an existing version of the APM and modified the scheduling process to use self-organising legitimate-claims CPR allocation. The data for the investigation of a simulation run with a predefined number of simulation time steps (ticks) are captured on different levels of observation and are aggregated accordingly:

System level:

- 1) The system's total residual load is assigned to and captured as **top-level AVPP residual load**.
- 2) The **total consumption-production gap** measures the actual deviation of total power, produced by all controllable power plants (CPPs), and power consumption (residual load). The consumption-production gap thus depicts the main measure for the system stability. For ease of comparison we define the *gap quotient* as $\text{gap}/\text{load}^{\text{res}}$.

AVPP level:

- 3) Statistical data of **satisfaction** capture aggregations of the satisfaction of an AVPP's constituents. Note that since scheduling is performed for a given number of lookahead steps in each time step, the data produced in each step are aggregated as averages.

In order to obtain statistically significant data, each *experiment*, i.e., each simulation with a specific parameter configuration, is repeated n times (*runs*). Thus, within an evaluation on system scope, two additional kinds of time aggregation are performed. Run aggregation combines the individual runs, computing the average and standard deviation for each value over all runs. Analogously, tick aggregation aggregates tick values to a single value for each experiment run.

Due to the huge parameter space, we empirically predetermined a suitable *standard parameter configuration* of **numerical experiment parameters** through a sensitivity analysis shown in Table III. Crucial parameters regarding satisfaction are the deviation thresholds ϵ^+ and ϵ^- for the satisfaction reinforcement. Higher statistic window sizes indirectly delay satisfaction reinforcement and thus stipulate a tolerance mechanism, whereas the reinforcement rates α and β stipulate direct control regarding satisfaction reinforcement.

In addition to the numerical parameters, the most important **categorical experiment parameters** are how the initial AVPP meso level hierarchy is determined and which type of claim-weight determination is used. The **initial hierarchy** determines the number of AVPPs and their compositions. It may either be a pre-defined structure or formed through a random structuration process in which power plants are randomly assigned to a random number of AVPPs. The set of physical power plants (PPPs), including their models, is pre-defined.

The AVPP hierarchy is technically modelled as a tree-structured partitioning graph (see Figure 1), with a top-level element denoted as *top-level AVPP*. Each AVPP has child elements which may either be AVPPs or PPPs, whereas the leaves have to be PPPs, i.e., CPPs or stochastical power plants (SPPs). We denote hierarchies consisting of only one layer of intermediary AVPPs as *flat*. The pre-defined initial flat hierarchy H_{flat} consists of 10 AVPPs and a total of 523 PPPs (350 SPPs, 173 CPPs). We use real power plant-, weather-, and load data from public sources for an area of Bavaria.

Table III. NUMERICAL EXPERIMENT PARAMETERS AND STANDARD PARAMETER CONFIGURATION

System	Subsystem	Parameter	Value
APM	Scheduling	lookahead steps	4
	APM LC allocator	statistics window size	10
	"	ω_d	0.2
	"	ω_{lc}	0.8
LPG'	deviation	ω^+	0.5
	satisfaction	ϵ^+	0.2
	"	ϵ^-	-0.2
	"	α	0.1
	"	β	0.1

We denote fairness as statistical dispersion of AVPP satisfaction, similar to [3]. The fairness metric is thus defined as the Gini inequality coefficient [17] $G(t) \in [0; 1]$ over the AVPP satisfactions $\sigma_a^A(t), a \in AVPP$. $G(t) = 0$ indicates maximum fairness at time step t , i.e., the satisfactions of all AVPPs are equal, whereas $G(t) = 1$ depicts minimum fairness, i.e., maximally dispersed AVPP satisfactions. The mean satisfaction is also given but is insufficient to describe fairness. If satisfaction is generally low, a distribution where a single agent has high satisfaction (indicated by a high Gini coefficient) is considered undesirable and vice versa. In general, high mean satisfaction and low Gini coefficient are thus ideal.

B. Scheduling Performance & Fairness Evaluation

We evaluate the performance of the LPG' scheduling on the scope of the system by comparing it to the CSOP-based scheduling firstly in terms of consumption-production gap and secondly by the resulting total satisfaction. To ensure comparability, two experiments operate on the same flat pre-defined initial hierarchy H_{flat} .

Figure 5 shows **mean residual load** and **mean production-consumption gap** time-curves for CSOP scheduling, where the CPLEX optimizer is used to solve the CSOP, and LPG' scheduling respectively. Both experiments are repeated $n = 10$ times and run for 500 time steps in each run. Since we use the typical scheduling interval of 15 minutes per time step, the residual load oscillates with a cycle length of 96 ticks, corresponding to 1 day. Because the residual load is the main input to the scheduling, its oscillation is reflected in the production-consumption gap, though only the cycle length is reflected. Also the gap cycle profiles are only qualitatively similar to each other. In the first half of a cycle, the mean gap quotient of CSOP scheduling is around 0%, whereas in LPG' scheduling it is considerably higher, around 4% (-50,000kW). Interestingly, their mean gaps are very similar to each other in the second part of a cycle. The average values of residual load, gap quotient and absolute gap over all ticks are presented in Table IV. The average gap of the LPG' scheduling is about 14 times higher than the CSOP scheduling gap. However, the average standard deviation of the mean gap quotient over the individual runs of 0.261% for the LPG' scheduling is only about half the value of CSOP. Comparing **satisfaction and fairness**, LPG' scheduling considerably outperforms CSOP scheduling with approximately 4.3 times higher mean satisfaction and 4.6 times higher fairness (Table IV, Figure 6). Overall the LPG' scheduling seems suitable to replace CSOP-based scheduling. Its lower performance regarding production-consumption gap was expected due to the sacrifice of CSOP's optimality.

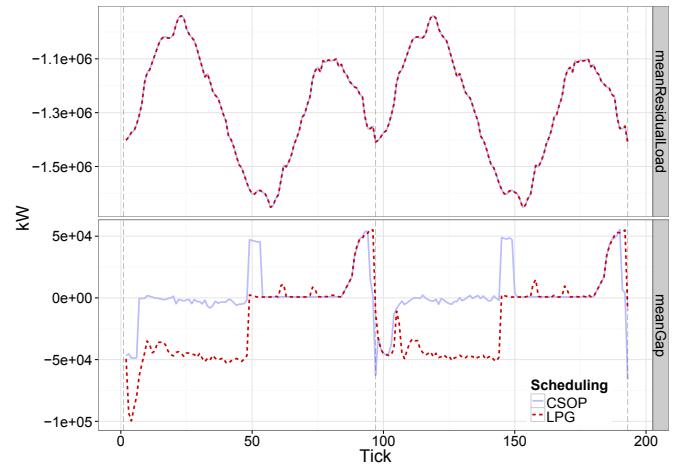


Figure 5. Residual load and consumption-production gap time-curves on the system scope (H_{flat} , run-aggregated). The mean consumption-production gap is higher in the LPG'- than in the CSOP scheduling.

Table IV. LOAD GAP AND AGENT SATISFACTION

	Tick Aggregation		Run Aggregation
	Mean	SDev	Average SDev
Residual load [kW]	-1,270,722	197,500	0
CSOP (H_{flat})			
Gap quotient [%]	0.124	3.332	0.508
Prod./Cons. gap [kW]	-1,398	45,441	6,119
Satisfaction	0.200	0.155	0.032
Gini coefficient	0.438	0.090	0.025
LPG' (H_{flat})			
Gap quotient [%]	1.717	3.843	0.261
Prod./Cons. gap [kW]	-20,199	49,916	3,121
Satisfaction	0.864	0.152	0.007
Gini coefficient	0.095	0.031	0.002
LPG' (rand.)			
Gap quotient [%]	1.807	3.861	0.350
Prod./Cons. gap [kW]	-21,233	49,968	4,266
Satisfaction	0.832	0.176	0.111
Gini coefficient	0.104	0.031	0.089

In a last experiment, we determined the AVPP structure randomly with a uniformly distributed random number of partitions, ranging from 1 (grand coalition) to 50. The sample size is $n = 50$. As shown in Table IV, the performance of LPG' scheduling is stable also for random initial structures, while the average standard deviation of the runs is increased, as expected due to varying initial structures.

C. Trade-Offs between Fairness and Utility

The evaluation results clearly show a trade-off between utility in form of the consumption-production gap and fairness, indicating the system's pareto optimality. Fairness is confirmed to come at a price as expected. We have focussed on the short-term effects of a fair allocation on the power plant's satisfaction in our evaluation. We did not explicitly model reactions of the agents to a perceived low fairness, such as abandoning the AVPP they are currently part of. In the short-term, such behaviour may be prohibited, e.g., by market contracts.

More interesting are a) the mid- and long-term implications of continuously dissatisfied participants and b) the attract-

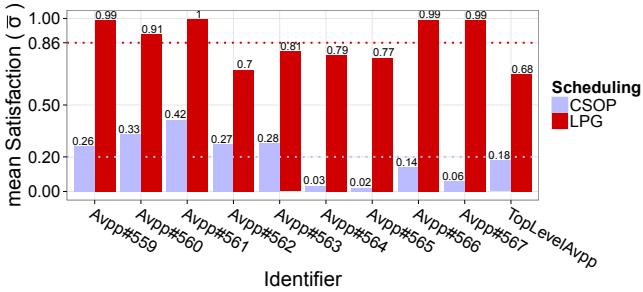


Figure 6. Illustration of run- and tick-aggregated satisfaction dispersion as explanation for respective Gini coefficients on the AVPP scope for CSOP and LPG' scheduling. Respective mean satisfaction values are indicated through dashed lines.

iveness of a scheme for new entrants. For the purposes of this discussion, an AVPP can be considered as a market in which load is allocated and compensation for this allocation is distributed. The higher the allocation, the higher the payout. The economic incentive for each participant to be part of the market is thus based on its ability to contribute. Power markets often reimburse power plant operators for providing the *possibility* of power production. This is not necessarily efficient since considerable funds are paid without receiving any actual power from the plants. A scheme based on fairness, however, guarantees that each power plant receives a share of the payments for power that is actually delivered to the system.

This approach incentivises operators to join the scheme and participate in the long-term. Such behaviour has a positive effect on stability and robustness since it ensures that sufficient potential is available in critical situations. By sacrificing a small quantity of utility, we gain a significant increase in total satisfaction and fairness, which gives all participants a strong incentive to continue contributing to the system. Moreover, the losses in utility are tolerable and can be compensated by the inertia of the system.

VI. RELATED WORK

The notion of Virtual Power Plant (VPP) has been introduced in the late nineties as a combination of locally dispersed power producers, and has been widely adopted by both science and industry visions since the number of Distributed Energy Resources (DERs) has started to grow significantly in the early 2000s. A VPP constitutes a set of logically and physically interconnected DERs, where their power productions are aggregated and represented by a single entity acting like a conventional power plant. Common goals of VPP approaches are the enhancement of the visibility of a group of DERs to allow for controllability and consideration in scheduling [18]. To maximise both collective and individual profit, VPPs are to enable power market access for groups of DERs [18].

Electronic markets have become a popular metaphor for cooperative algorithms that solve the scheduling problem. DEZENT [19] is used to balance energy supply and load in a hierarchical system structure in a bottom-up manner. A further approach based on a hierarchical system structure is PowerMatcher [20] in which the root of a tree balances supply and load by determining an equilibrium price, based on aggregated load, supply, and price predictions, to establish a market equilibrium. Fairness is not considered in either of these

approaches, thus making it possible that individual participants are effectively shut out of the market. Stigspace [21] is a coordination mechanism that uses a blackboard, called stig-space, as the medium of communication between distributed energy resources in order to create schedules in an iterative process. Again, fairness is not considered, allowing some power plants to satisfy all the load and shutting out others. The same distinction holds w.r.t. TruCAOS [1], the cooperative scheduling mechanism introduced in Section I.

Incentivising participants to stay in the system and participate in the market is especially important when the allocation of resources is repeated over time. In such cases, the system benefits from having as many participating agents as possible. One way of providing such incentive is through fairness. Fairness has been well studied in the area of communication networks. However, the main goal of having fair allocations in this case is to improve the system's performance, usually through load balancing and flow control [22], and not that of making the agents 'happier'. Besides, in this domain, there is an absolute control of who can and cannot use the resources at any given time. On the other hand, in systems such as power management and other open systems, agent satisfaction becomes relevant. While a fair allocation might not be economically optimal [2], it has indirect benefits such as increasing the satisfaction of the participating agents, which in the long-run might actually be better than the classical 'optimal' solution. This kind of benefits have already been shown in the domain of auctions, in particular for service oriented marketplaces [23], [24]. These works address, among others, the *bidder drop problem*, in which those agents repeatedly left out of any allocation leave the marketplace. As a consequence, the remaining agents gain more power for setting higher prices, which in the long-term can lead to the collapse of the system due to excessive cost. Power management systems face a similar problem, whereby those plants not being allocated any production might decide to leave, thus leaving the system with fewer available plants, which as we already mentioned can be highly problematic.

VII. CONCLUSION & OUTLOOK

This paper combines concepts of fairness in electronic institutions with algorithms for dynamic resource allocation in power management systems. We started out creating a fair resource allocation mechanism and in the course of the endeavour uncovered a deeper principle: fairness and individual satisfaction of the agents are secondary optimisation criteria that should be considered in addition to standard utility optimisation to ensure robust, resilient, and enduring organisations. The contributions of the paper are as follows:

- We propose a novel approach to resource allocation that is a convergence of electronic institution and trust and cooperation for dynamic allocation algorithms. The approach considers issues of fairness and individual agent satisfaction as well as solution utility.
- We illustrate how such an approach can be applied to the domain of autonomous power management. This domain benefits especially from the aspects we focus on due to the open nature of such systems and the requirement to include all power plants in the power production.
- An extensive testbed for the evaluation of the algorithms, of agent satisfaction, and overall utility is laid out that al-

lows creating experiments with different system structures and power plant configurations.

- The experimental results show that there is a trade-off between utility and fairness: increasing the latter comes at the price of decreasing the former, and vice-versa.
- Combining our results with those shown in [3], we can turn the *price* of fairness into *value*, since its consideration in dynamic resource allocation contributes to the optimisation of qualitative objectives such as robustness, resilience and endurance.

Our results are of utmost importance, since we are observing a transition from closed systems with an abundance of resources and no choice to open systems with a scarcity of resources and a choice of who these resources are allocated to. Power management systems are one exemplar of a complex system in which such a transition is currently taking place. There is a move away from centralised production and control and a small number of powerful generators in the hand of one utility towards distributed generation that includes generators from a number of companies and private citizens. For them, fair resource allocation and thus fair allocations of revenue is a necessity—otherwise, there would be no incentive to participate in the system.

The presented approach contrasts with “classical” resource allocation literature, where only utility—a quantitative objective—is considered. Criteria such as robustness, resilience, and endurance are, however, not directly measurable and only manifest in the long-term and in moments of crisis. Since we showed how to measure fairness, and fairness contributes to these criteria, we have also provided a way to at least indirectly include these considerations in strategic planning.

Future work will further investigate these relationships in long-term experiments with heterogeneous agents that value satisfaction differently. We will study how these agents react to disturbances and how the stability of the system compares to “classical”, non-fair resource allocations. Furthermore, we will study the inclusion of the concept of *social capital* in electronic institution: power plants could accrue social capital by compensating unintentional errors of other power plants. This capital can then be included in claims or be reciprocally rewarded by the malfunctioning power plants in other situations. An interesting aspect of our work is the potential feedback of agent satisfaction into the self-organisation of agent structures. An institution’s dissatisfied agents can decide to abandon the current structure and create their own organisation. On the other hand, a successful organisation can attract other agents. Of course, there is a fine line between system stability—the goal we try to achieve with the inclusion of fairness—and optimising the system structure towards agent satisfaction. In the worst case, the two adaptation mechanisms could interfere negatively, in the best case they reinforce each other.

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